Technical Comments

Comment on "Spanwise Distribution of Induced Drag in Subsonic Flow by the Vortex Lattice Method"

G. J. HANCOCK*
Queen Mary College, University of London, England

A REACTION to Ref. 1 might be of some interest. The first point concerns the over-all philosophy of approach. To calculate the induced drag the authors determine the downwash at the midpoint on each bound vortex element and then sum the induced drags (downwash × bound vortex strength) for all elements. It is found that the total induced drag is underestimated compared with the exact, Trefftz plane, value. Although the vortex lattice method leads to a finite lift, strictly it implies an infinite induced drag, because the induced drag of each single horseshoe vortex line is itself infinite. The authors bypassed this point without comment by their approximate technique. It might be argued that the degree of approximation inherent in the calculation of the induced drag in Ref. 1 is consistent within the general context of the vortex lattice method. But such an argument requires further examination. The success of the vortex lattice method depends on the fact that the downwash condition is satisfied on the average over the wing surface; in effect the infinities in downwash across all vortex lines tend to cancel out. But this argument does not hold for the induced drag since the infinities in induced drag across trailing vortex lines do not tend to cancel out, downwash velocities are equal and opposite on either side of a trailing vortex line but the strengths of the bound vorticity differ. What is fairly clear, however, is that the method of calculation in Ref. 1 gives the lowest estimate for the induced drag since the downwash at the midpoint on a bound vortex line is a minimum. Reference 1 shows therefore that this minimum estimate of total induced drag is not too far removed from the exact value; it might be anticipated that, apart from the value of the total induced drag, the spanwise distribution over the wing should be reasonable.

The second point is somewhat more fundamental. A defect in Ref. 1 is that there is no explanation of why the authors calculated the induced drag in the manner that they did. Admittedly the purpose of the paper is directed towards the spanwise distribution of induced drag but there is an implied chordwise distribution. And this chordwise distribution is incorrect. To go back to first principles, the induced drag has two contributions; one arises from the inclination of each elemental lift vector, through the inclination of each element to the stream direction, while the other is the negative contribution from the leading-edge thrust. Reference 1 does not distinguish between these two effects. If a more fundamental approach had been adopted, then an alternative method of calculation of the induced drag from the vortex lattice model would have been

$$\text{Induced drag} = \sum_{\substack{\text{all}\\ \text{elements}}} [(\text{elemental load}) \times (\text{elemental streamwise inclination})] - \frac{\pi}{32} \sum_{\substack{\text{leading-}\\ \text{edge}\\ \text{elements}\\ \text{only}}} [(\text{elemental load})^2/\tfrac{1}{2}\rho V^2(\text{elemental area})] \quad (1)$$

Equation (1) is a more direct method than that described in Ref. 1, since it does not include the calculation of any further downwash velocities.

Equation (1) leads to an insight into the method outlined in Ref. 1, where the formula for the induced drag is basically

$$\text{Induced drag} = \sum_{\substack{\text{all}\\ \text{elements}}} \{(\text{elemental load}) \times (\text{incidence at } \tfrac{1}{4} \text{ point})\} \quad (2)$$

The difference between Eqs. (1) and (2) shows that, by taking different downwash velocities on each element, Eq. (2) essentially redistributes the concentrated leading-edge thrust force over the entire chord. It would be instructive to compare Eqs. (1) and (2) and check whether or not Eq. (1) leads to any improvements.

A specific reason for suggesting that the method of calculation should follow from the application of Eq. (1) is prompted by the throw away statement in the last paragraph in Ref. 1 which states that the method outlined in Ref. 1 can be used for certain quasi-steady rotary derivatives. Without further proof this statement is unacceptable. For example, consider the simple problem of the induced drag on a rolling wing. Now a rolling wing has only an induced thrust because the element inclination to the stream direction is zero, so from Eq. (1) only the leading-edge thrust contribution remains; this is an example where the elemental inclination is not identical to the elemental incidence. Thus, the method outlined in Ref. 1, as expressed in Eq. (2) would give completely the wrong answer; to start with, even the sign would be wrong.

The authors of Ref. 1 have amply demonstrated in their many publications that the vortex lattice method is a powerful and reliable tool; it is hoped that the above remarks help clarify some of the fringe applications.

Reference

[1] Kalman, T. P., Giesing, J. P., and Rodden, W. P., "Spanwise Distribution of Induced Drag in Subsonic Flow by the Vortex Lattice Method," *Journal of Aircraft*, Vol. 7, No. 6, Nov.–Dec. 1970, pp. 574–576.

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* Staff Member, Department of Aeronautical Engineering.

Reply by Authors to G. J. Hancock

T. P. KALMAN,* J. P. GIESING,† AND W. P. RODDEN‡
Douglas Aircraft Company, Long Beach, Calif.

THE vorticity distribution on any lifting surface is continuous. A discontinuity in vorticity would cause an infinite induced drag. In our Note this continuous distribution is replaced by a vortex-lattice system. As with many numerical approaches, this replacement system is valid only at a specific set of collocation points. For lift and moment calculations, the $\frac{3}{4}$ chord point on the spanwise centerline of each element must be used. This same collocation point could be used for drag if the leading-edge suction could also be calculated directly as in Hancock's Eq. (1). James[1] has shown for a large number of chordwise elements that, in the

* Engineer/Scientist, Structural Mechanics Section. Member AIAA.
† Senior Group Engineer, Structural Mechanics Section. Associate Fellow AIAA.
‡ Consulting Engineer. Associate Fellow AIAA.

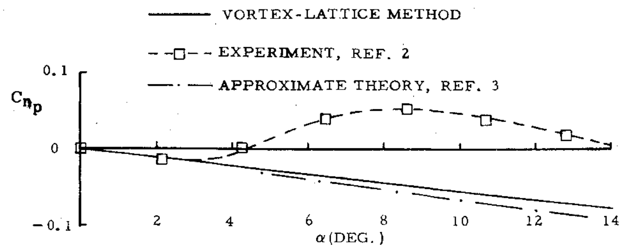

Fig. 1 Correlation of calculated and measured values of yawing moment coefficient due to the rolling.

two-dimensional incompressible case and for constant downwash, the pressure calculated on the leading-edge element by the Vortex-Lattice Method is always low by 11.4%. On the basis of James' investigation, it could be anticipated that Hancock's Eq. (1) would be inaccurate. The magnitude of the inaccuracy is indicated by the example of the hyperbolic wing in our Note: the Vortex-Lattice estimate of the total induced drag was low by 8% while Hancock's Eq. (1) overestimated the drag by 64%; the example had five elements on the chord. An alternate approach was to apply the Kutta-Joukowski Law. Such an application made directly to the replacement vortex-lattice system would give an infinite induced drag since the replacement lattice is not a continuous surface vorticity system. Again a set of collocation points must be selected for the vortex-lattice replacement system: hence, Hancock's Eq. (2) [our Eq. (4)]. As the results of our Note show, the collocation point located at the $\frac{1}{4}$-chord location on the spanwise centerline of each element gives good results.

Hancock is correct in stating that the leading-edge suction obtained by the Vortex-Lattice Method is distributed over the chord. Actually it is distributed over the first few forward elements. As the number of elements is increased, however, the leading-edge suction moves to the leading edge. The purpose of our Note was to predict the spanwise distribution of induced drag and only as many chordwise elements were used as affected the convergence of the spanwise distribution. If the distribution of leading-edge suction coefficient is needed for some reason, it can easily be determined from the Vortex-Lattice Method as the difference between the drag component of the local normal force coefficient and the local induced drag coefficient.

On the subject of induced drag for rolling motions, Hancock should not have inferred that his Eq. (2) [our Eq. (4)] is sufficient in itself to calculate rotary derivatives since the Kutta-Joukowski Law also introduces a horizontal component of the lift vector in the stability axes coordinate system in addition to the drag component. The complete expression for the yawing moment coefficient because of the rolling is

$$C_{n_p} = \frac{1}{2}\int_0^1 \left[\frac{1}{2}\,\frac{\Delta c_d}{pb/2V} - \bar{c}_l\eta\right]\frac{c}{\bar{c}}\,\eta d\eta \qquad (1)$$

where η is the semispan fraction, $c/\bar{c}$ is the ratio of local to average chords, Δc_d is the difference between right and left wing drag coefficients, $\bar{c}_l$ is the average between right and left wing lift coefficients, and $pb/2V$ is the rolling helix angle. The rolling helix angle is taken to be small in order to obtain the linearized rotary derivative. The lift and drag coefficients are calculated for a downwash consisting of a steady roll added to the angle of attack

$$w = (pb/2V)\eta + \alpha \qquad (2)$$

Experimental data have been presented by Wiggins[2] for rolling derivatives at angles of attack up to 13° and at high subsonic Mach numbers. The calculation of Eq. (1) for the planform with 32.6° sweep of the quarter chord, an aspect ratio of 4.0 and taper ratio of 0.6 resulted in $C_{n_p}/\alpha = -0.005538$/deg at a Mach number of 0.7. The calculation chose $(pb/2V)/\alpha = 0.01$ and $\alpha = 0.12$ rad. The Vortex-Lattice result is compared to the experimental data in Fig. 1 and is seen to predict the initial slope of the curve both in *sign* and magnitude; the nonlinearity caused by viscous effects on leading-edge suction at higher angles of attack, of course, could not be predicted, but the present results can be regarded as a basis for applying the semiempirical method of Wiggins[2] to account for viscious effects. An approximate calculation for C_{n_p}/α for low speed taken from [Ref. 3, Eq. (31), Page 18] gives a value of -0.00665/deg. This calculation is also shown on Fig. 1.

References

[1] James, R. M., "On the Remarkable Accuracy of the Vortex Lattice Discretization in Thin Wing Theory," Rept. DAC-67211, Feb. 1969, Douglas Aircraft Co., Long Beach, Calif.

[2] Wiggins, J. W., "Wing-Tunnel Investigation of Effect of Sweep on Rolling Derivatives at Angles of Attack up to 13° and at High Subsonic Mach Numbers, Including a Semiempirical Method of Estimating the Rolling Derivatives," TN 4185, Jan. 1958, NACA.

[3] Toll, T. A. and Queijo, M. J., "Approximate Relations and Charts for Low-Speed Stability Derivatives of Swept Wings," TN 1581, May 1948, NACA.